MODELING OF NONLINEAR SYSTEMS USING GENETIC ALGORITHMS

Alaa Sheta
Computers and Systems Department
Electronics Research Institute
Cairo, Egypt
asheta1@eri.sci.eg

Sultan H. Aljahdali
School of Information Technology
George Mason University
Fairfax, VA 22030, USA
saljahdali@gmu.edu

ABSTRACT

This paper demonstrates the ability of Genetic Algorithms (GAs) in the identification of dynamical nonlinear systems. The dynamics of the nonlinear systems have been described by first, second and third order terms. GAs were used successfully to identify the coefficient of these terms. A comparison between least-square estimation (LSE) and genetic algorithms estimation (GAE) procedures is provided. The comparison was employed based on two factors, number of observations and estimation accuracy. Genetic algorithms show better performance in both noise free and noisy cases.

KEY WORDS

Modeling, Nonlinear Systems, Genetic Algorithms

1. INTRODUCTION

The complexity of modern control system technology and its corresponding associated problems causes difficulties in the analysis and design for such systems using traditional theories. System identification and parameter estimation is one of the tremendously affected techniques. The reason is that the assumption imposed on the calculation and results of the model identification process are often not immune to criticism or based on real prior knowledge. For example, in the identification process using least square technique an information about the noise imposed on the observations need to be known. Using maximum likelihood an assumption about the probability density function (pdf) usually applied.

The motivation behind this work is to explore the ability of GAs in handling the identification problem for dynamical nonlinear systems using a limited number of observations and in lack of *a priori* knowledge about the noise. Also, we will compare the results for both LSE and GAE procedures in the same environmental conditions.

We have developed different kinds of models, first, second and third order models as a function of the system input and the system output. The structure of these models are sort of the series-parallel models presented in [1]. We are assuming that the nonlinear systems are unknown and all available information is a limited set of input-output observations. These observations will be considered as the base of testing and evaluating the considered models.

The difference between genetic algorithm and least square technique is that GAs are a stochastic search while LS technique is a gradient search. The former are able to explore the error surface in a way that they do not stuck by local optimum. The later is usually function of the system initial estimate so local optimal might be reached. Besides, it is highly affected by noise.

2. PREVIOUS WORK

Genetic algorithms (GAs) have been introduced in 1975 by J. Holland [2]. De Jong [3] have explored different characteristics of GAs. GAs have been recognized by the control community [4, 5, 6, 7, 8]. In the past few years, there were a great interest of using GAs to solve difficult control problems. For example, it was used in the design of a digital proportional integral differential (PID) controller [9, 10]. They were valuable in parameter estimation of nonlinear systems [11, 12, 13], multi-objective optimization [14], robust control design of an aircraft [15], system integration [16], adaptive control [17], and observer design [18]. They were also used in real-life control application [19]. This list can be extended.

3. IDENTIFICATION METHODOLOGY

Assume we have a nonlinear dynamical system described by the following characteristics:

$$y(k+1) = \phi[y(k), u(k)]$$

$u(k)$ and $y(k)$ are the system input and output, respectively. In this study, we are assuming that the function ϕ is unknown. Thus, our goal is to describe a suitable model structure for ϕ as a function of $u(k)$ and $y(k)$ such that the error difference between the actual system $y(k)$ and the estimated model $\hat{y}(k)$ responses is minimum.

In [1] Narendra has explored the usage of neural network to identify the dynamics of nonlinear systems using a proposed series-parallel models. These models have the following characteristics:

$$\hat{y}(k+1) = \phi[y(k), y(k-1)] + N[u(k)]$$

Neural networks have been proposed to simulate the structure of the functions ϕ and N. Unfortunately, they still do not give any idea about the possible structure of the nonlinear systems. A certain equation that describe the relationship between the system input $u(k)$ and output $y(k)$ need to be identified. The above model structure will be considered as a guidance for our modeling process.

We have described different functions that we believe will best simulate the nonlinear system behavior. This believe is based on an experimental effort in the development of these functions. Genetic algorithms are used to identify the parameters for the proposed function structure. A 100 observations uniformly distributed between -2 and 2 were used in the identification process. An input test sequences used in [1] will be used to test the behavior of the developed models. LSE and GAE procedures are employed to identify the values of the parameters for those models. A calculation for the standard deviation of the error in both cases will be considered as the major of success.

In the following section, we will describe three examples presented in [1]. Also, show that GAs are performing better than LSE in solving the identification problem for dynamical nonlinear systems using a limited number of observations. Our method has the advantage over neural network because we are providing a model structure that can be used for prediction and control purposes.

3.1 SYSTEM WITH NONLINEAR INPUT

The system to be identified is described by the following difference equation:

$$y(k+1) = 0.3y(k) + 0.6y(k-1) + f[u(k)]$$

The function f has the following form $f(u) = 0.6sin(\pi u) + 0.3sin(3\pi u) + 0.1sin(5\pi u)$. To identify the given system a series-parallel model governed by the difference equation

$$\hat{y}(k+1) = \hat{\alpha}_1 y(k) + \hat{\alpha}_2 y(k-1) + N[\alpha, u(k)]$$

is used. In our case, we will define the function $N[\alpha, u(k)]$ as follows:

$$N[\alpha, u(k)] = \hat{\alpha}_3 u(k) + \hat{\alpha}_4 u^2(k) + \hat{\alpha}_5 u^3(k)$$

The input signal $u(k)$ is presented by a sequence of random numbers uniformly distributed between -2 and 2. The parameters $\hat{\alpha}$ for the proposed model were estimated using a limited number of observations using batch LSE algorithm and GAE algorithm. The estimated values of the parameters α were:

$$\hat{\alpha} = (0.279, 0.601, 0.745, 0.022, -0.361)$$

In Figure 1, we show the system response for both LSE and GAE procedures. The standard deviation of the error in LSE and GAE were 0.2635 and 0.1372, respectively. GAs were able to achieve better estimation values for the model parameters than LSE, with a small number of observations.

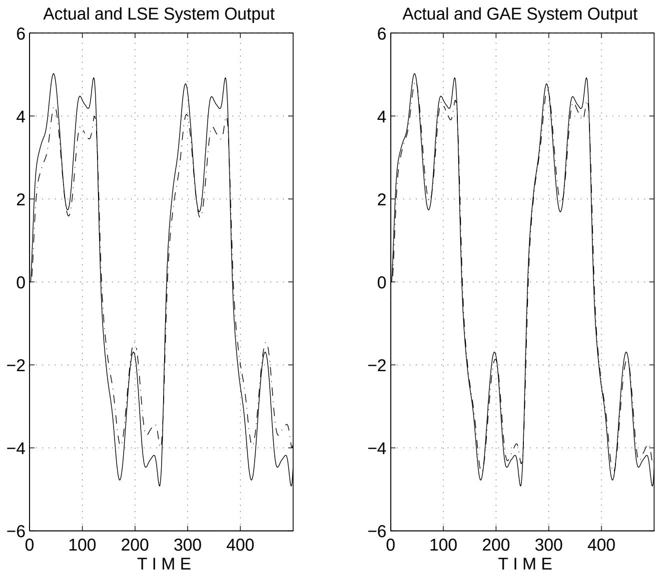

Figure 1. System and model output responses using LSE and GAE process for system 1

3.2 SYSTEM WITH NONLINEAR OUTPUT

The system which has a nonlinear output is governed by the second-order difference equation:

$$y(k+1) = \phi[y(k), y(k-1)] + u(k)$$

where:

$$\phi[y(k), y(k-1)] = \frac{y(k)y(k-1)[y(k)+2.5]}{1 + y^2(k) + y^2(k-1)}$$

The proposed model was considered as follows:

$$\begin{aligned}\hat{y}(k+1) &= \hat{\alpha}_1 y(k) + \hat{\alpha}_2 y(k-1) \\ &+ \hat{\alpha}_3 y^2(k) + \hat{\alpha}_4 y^2(k-1) \\ &+ \hat{\alpha}_5 y(k) y(k-1) + \hat{\alpha}_6 y(k) y^2(k-1) \\ &+ \hat{\alpha}_7 u(k)\end{aligned}$$

The values for the estimated parameters $\hat{\alpha}$ were:

$$\hat{\alpha} = (0.104, 0.191, 0.046, 0.031, 0.351, -0.074, 0.991)$$

The standard deviations of the error in the cases of LSE and GAE were 0.2505 and 0.1295, respectively. In Figure 2, we show the actual and estimated system responses in both cases.

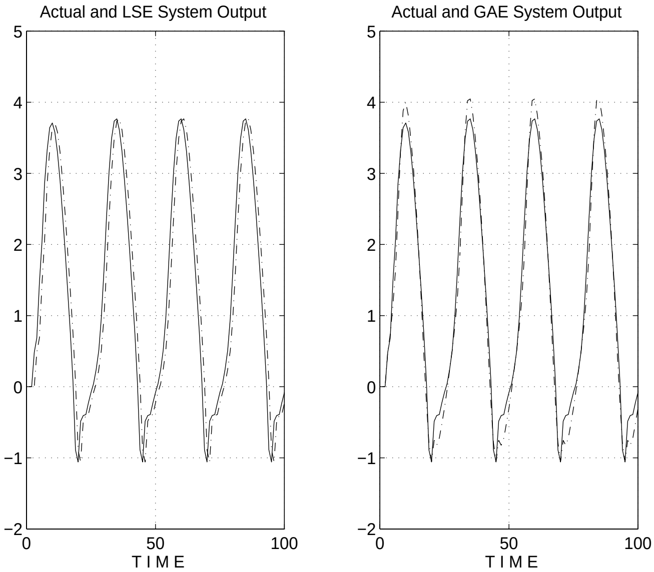

Figure 2. System and model output responses using LSE and GAE process for system 2

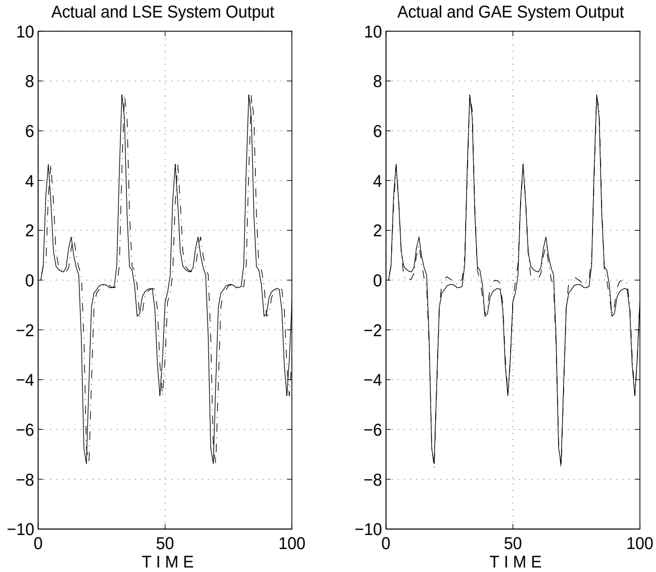

Figure 3. System and model output responses using LSE and GAE process for system 3

3.3 SYSTEM WITH NONLINEAR INPUT AND OUTPUT

In this system we consider both nonlinearity in the system input and output:

$$y(k+1) = f[y(k), y(k-1)] + u^3(k)$$

where:

$$f[y(k), y(k-1)] = \frac{y(k)}{1+y^2(k)}$$

For system 3 we have described a model has the following characteristics :

$$\hat{y}(k+1) = \phi[\hat{\alpha}, y(k)] + \psi[\hat{\beta}, u(k)]$$

An identifier is used to identify the plant from input-output data and is described by the equation

$$\begin{aligned} \hat{y}(k+1) &= \hat{\alpha}_1 y(k) + \hat{\alpha}_2 y^2(k) \\ &+ \hat{\beta}_1 u(k) + \hat{\beta}_2 u^2(k) + \hat{\beta}_3 u^3(k) \end{aligned}$$

The estimate model parameters were:

$$\hat{\alpha} = (0.046, 0.002), \hat{\beta} = (0.121, -0.019, 0.971)$$

In Figure 3, we show the actual and estimated system responses for LSE and GAE procedures. The standard deviation of the error in LSE and GAE were 1.1929 and 0.1305, respectively.

4. IDENTIFICATION IN THE PRESENCE OF NOISE

Manipulating the identification process in noisy environment is a challenge choice. The reason is, when noise is added to the observations, a completely different error surfaces will be created. The noise can be added on the input or output response. The former is a simple job since the more the system excited the better the performance of the estimated parameters. The later is a difficult one since noisy observations makes the identification process quite difficult job. Our aim her is to test the ability of the genetic algorithms to operate in a very high noisy environment and apply the same identification procedure as in the previous section. We will define a signal to noise ratio (SNR) as a measure for the noise level. The definition of SNR will be the ratio between the standard deviation of the signal to the standard deviation of the noise:

$$SNR = \frac{Std(Signal)}{Std(Noise)}$$

4.1 SYSTEM WITH NONLINEAR INPUT

Applying the identification process for System 1 with noisy observations. The SNR in this case was defined as 3.3. We have used the same excitation sequence used in the noise free case. This signal is a random sequence uniformly distributed between -2 and 2. The same tuning parameters for GAs have been used as population size, crossover and mutation. The values of the estimated parameters in this case were :

$$\hat{\alpha} = (0.292, 0.612, 0.455, 0.011, -0.2480)$$

In Figure 4, we show the actual system response and the estimated GA response while testing the system by an input signal $u(k) = sin(2\pi k/250)$. In this case the standard deviation of the error was 0.1358.

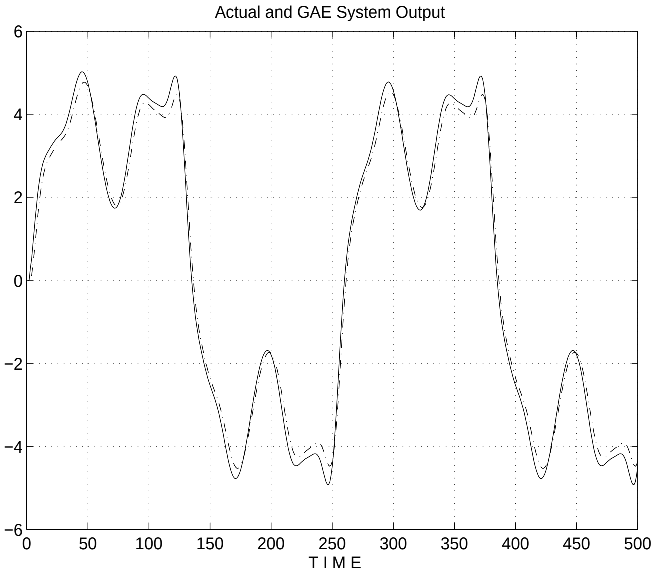

Figure 4. System and model output responses using LSE and GAE process for system 1

4.2 SYSTEM WITH NONLINEAR OUTPUT

For system 2 the SNR used was 10. The values of the estimated parameters was:

$$\hat{\alpha} = (-0.02, 0.088, 0.106, 0.092, 0.338, -0.083, 0.9540)$$

The standard deviation of the error was 0.1614. The system was tested by an input signal $u(k) = sin(2\pi k/25)$. In Figure 5, we show the actual system response and the estimated GA response.

4.3 SYSTEM WITH NONLINEAR INPUT AND OUTPUT

For system 3, the SNR was selected as 3.5. The values of the estimated parameters were:

$$\hat{\alpha} = (0.016, -0.002), \hat{\beta} = (0.143, -0.036, 1.0)$$

The standard deviation of the error was found as 0.1522. In Figure 6, we show the actual system response and the estimated GA response while testing the system by an input signal $u(k) = sin(2\pi k/25) + sin(2\pi k/10)$.

From the above presented results it can be seen that GAs were able to identify the dynamics of nonlinear systems from noise free and noisy observations successfully.

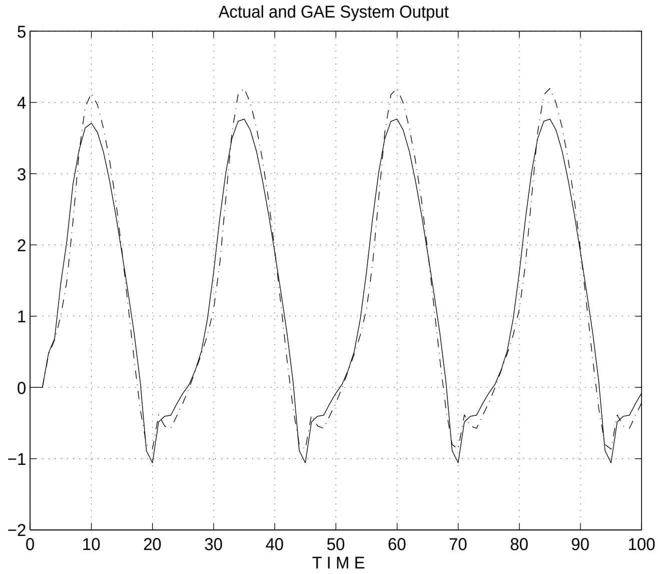

Figure 5. System and model output responses using LSE and GAE process for system 2

The standard deviations of the error in both cases were very small. The difficulties arises when noise is presented in the observation is that; GAs will need a larger number of evaluation (i.e, more generations to run). The models described above were developed for these systems specifically. For other systems we need to develop different model structures. The robustness of the described models, as in any identification procedure, depends on the input excitation signal. Thus the better excitation signal applied to the system the better is the estimated values of the model parameters. Our intuition was to follow the same experiments presented in [1]. It is to know that the parameter estimation using LSE in noisy environment will provide poor results.

5. CONCLUSION

In this paper, we have used genetic algorithms in the identification of nonlinear systems. Genetic algorithms are stochastic, adaptive and robust search algorithms successfully able to work in unnatural environments regarding the noise. We used GAs to estimate the parameters of the developed models that can fit to nonlinear systems. This methodology has few advantages over other identification techniques like neural networks. First, we need less number of observations. Second, we have developed model structure which is useful in prediction and control. Third, we can check system stability and robustness based on the proposed models.

6. ACKNOWLEDGMENT

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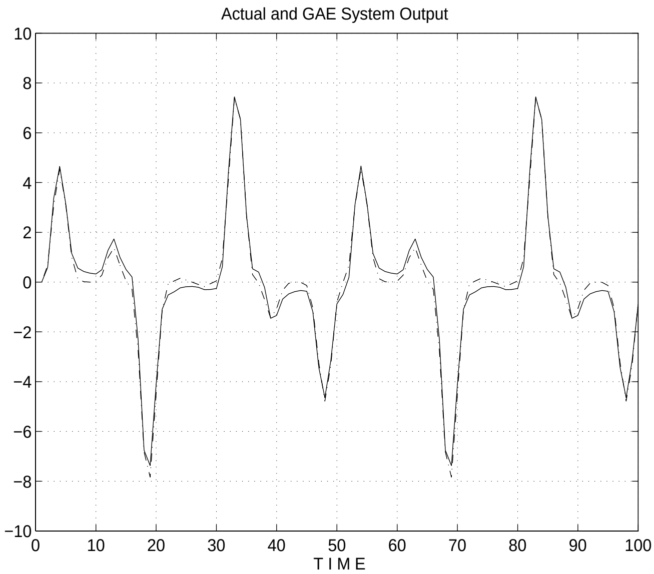

Figure 6. System and model output responses using LSE and GAE process for system 3

References

[1] K. Narendra and K. Pathansarathy, "Identification and control of dynamical systems using neural networks," *IEEE Transaction on Neural Networks*, vol. 1, 1990.

[2] J. Holland, *Adaptation in Natural and Artificial Systems*. Ann Arbor, MI: University of Michigan Press, 1975.

[3] K. A. De Jong, *Analysis of Behavior of a Class of Genetic Adaptive Systems*. PhD thesis, University of Michigan, Ann Arbor, MI, 1975.

[4] K. Kristinsson and G. Dumont, "System identification and control using genetic algorithms," *IEEE Transaction on Systems, Man, and Cybernetics*, vol. 22, no. 5, pp. 1022–1046, 1992.

[5] K. Passino, "Intelligent control for autonomous systems," *IEEE Spectrum*, 1995.

[6] A. J. Chipperfield and P. J. Fleming, "Genetic algorithms in control systems engineering," *IASTED Journal of Computers and Control*, vol. 24, no. 1, 1996.

[7] A. Chipperfield, P. J. Fleming, and C. Fonseca, "Genetic algorithms tools for control systems engineering," in *Proceedings of First International Conference on Adaptive Computing in Engineering Design and Control*, pp. 128–133, UK, 1994.

[8] J. Claverie, K. De Jong, and A. Sheta, "Robust nonlinear control design using competitive coevolution," in *Proceedings of the Congress on Evolutionary Computation (CEC2000)*, pp. 403–409, 2000.

[9] W. R. Hwang and W. E. Thompson, "An intelligent controller design based on genetic algorithms," in *Proceedings of the 32 International Conference on Decision and Control*, pp. 1266–1267, 1993.

[10] B. Porter and A.H.Jones, "Genetic tuning of digital PID controllers," *Electronics Letters*, vol. 28, no. 9, pp. 843–844, 1992.

[11] A. Sheta and K. DeJong, "Parameter estimation of nonlinear systems in noisy environment using genetic algorithms," in *Proceedings of the IEEE International Symposium on Intelligent Control (ISIC'96)*, pp. 360–366, 1996.

[12] A. F. Sheta, *Identification and Control of Dynamical Systems Using Genetic Algorithms*. PhD thesis, George Mason University, Fairfax, VA, 1997.

[13] M. E. El-Telbaney, A. Sheta, and A. Abdelwahab, "Structure determination and parameter identification using evolutionary algorithms," in *Proceedings of the International Conference on Soft Computing, Brno, Czech Republic, June*, 1999.

[14] C. M. Fonseca and P. J. Fleming, "Multi-objective optimal controller design with genetic algorithms," in *Proceedings of IEE Control*, pp. 745–749, 1994.

[15] R. J. Patton and G. P. Liu, "Robust controller design via eigenstructure assignment, genetic algorithms and gradient-based optimization," in *Proceedings of the IEE Control Theory Appl.*, vol. 141, 1994.

[16] I. C. Parmee and M. J. Denham, "The integration of adaptive search techniques with current engineering design practice," in *Proceedings of the 1st International Conference Adaptive Computing in Engineering Design and Control*, pp. 1–13, UK, 1994.

[17] L. Porter II and K. Passino, "Genetic model reference adaptive control," in *Proceedings of the IEEE International Symposium on Intelligent Control*, pp. 219–224, 1994.

[18] L. Porter II and K. M. Passino, "Genetic adaptive observers," *Engng Applic. Artif. Intell.*, vol. 8, no. 3, pp. 261–269, 1995.

[19] A. J. Chipperfield and P. J. Fleming, "Multi-objective gas turbine engine controller design using genetic algorithms," *IEEE Transaction Ind. Electronics*, 1996.